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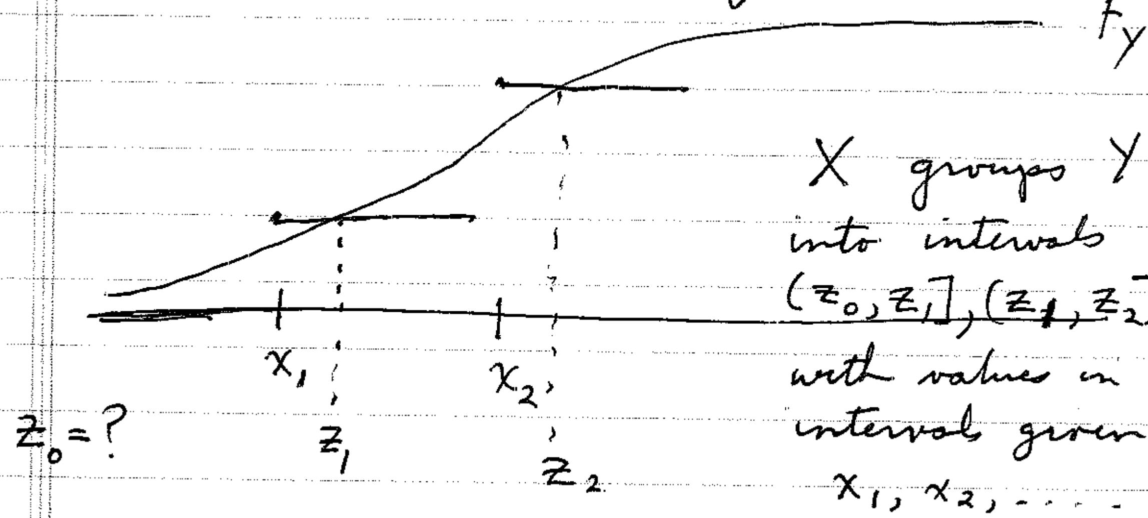
Some Observations from Last Week

Framework:

- Univariate Pop Y
- Approximate Y by X where
  - X has k support point
  - $E(X^j) = E(Y^j) \quad j = 1, \ldots, 2k-1$

$$X \overset{k}{\preceq} Y \iff E(f(X)) \le E(f(Y))$$

where f is k-convex.



X groups Y into intervals $(z_0, z_1], (z_1, z_2] \ldots$ with values in these intervals given as $x_1, x_2, \ldots$

A Digression: $X \overset{k}{\preceq} Y$ is a marginal concept. For technical reasons I want to ~~have~~ construct a joint $(X', Y') \ni X' \overset{d}{=} X$, $Y' \overset{d}{=} Y$ where the joint distribution has a "meaningful" structure between X' & Y' that accounts for using X to approximate Y. I will call this "constructive probability" since the construction of the joint is done to account for some special structure.

Examples: (1) de Finetti's Theorem

Special Structure - infinite exchangeable sequence

$\{0,1\}$ exchangeable r.v.'s $\Leftrightarrow$ $X_1, X_2, \ldots | \mathcal{E}$ are iid Bernoulli

$\mathcal{E} \equiv$ exchangeable σ-field.

Proved by moment sequence arguements (Feller Vol II) or reverse martingale

(2) Row Column Exchangeable Array (Aldous)

$$X_{ij} = f(M, R_i, C_j, \varepsilon_{ij}) \quad \text{- Reverse Martingale}$$

Fubini

(3) Load Sharing Rules (LS)

- independent Weibull component $X_1, \ldots, X_k$
- dynamic system where as components fail load is transferred to unfailed components. Based on L.S

$$R(t) = P\left(\frac{S}{\Lambda} > t\right) \quad \text{where } S \perp\!\!\!\perp \Lambda$$

Based on $X_1, \ldots, X_k$

(4) Extreme values of Order Stats for Exponential

$$X_{k,n} = S_k / \Lambda_{k,n} \qquad S_k \perp\!\!\!\perp \Lambda_{k,n}$$

complicated but closed form.

gamma shape k

<u>In our problem</u> $X \overset{k}{<} Y \Leftrightarrow$

k-mart ~~structure~~ construction given in (★).

(★) $X_1, X_2, \ldots, X_k$ iid $\sim X$ with

$E(Y^j | X_1, \ldots, X_k) = \sum_1^k X_i^j / k$ (comparison of experiment)

$j = 1, \ldots, 2k-1$.

($k=1$, a one-mart, Blackwell, Stein, Sherman, Strassen)

<u>Q</u> How can I predict Y from X?

<u>A</u> Let $j = 1, \ldots, 2k-1$. Then,

$$E(Y^j | X_1) = E(E(Y^j | X_1, \ldots, X_k) | X_1)$$

$$= E\left(\frac{\sum_1^k X_i^j}{k} \,\Big|\, X_1\right) = \frac{X_1^j}{k} + \frac{1}{k}\sum_2^k E(X_i^j | X_1)$$

$X_2, \ldots, X_k \perp\!\!\!\perp X_1$

$$\overset{\downarrow}{=} \frac{X_1^j}{k} + \frac{k-1}{k}\left(E(X_1^j)\right) \leftrightarrow \text{smoothing formula.}$$

$$= E(X_1^j) + \frac{X_1^j - E(X_1^j)}{k}. \leftrightarrow \text{adjustment/control formula.}$$

Above, we have specified $2k-1$ conditional moments for $Y_1 | X_1$. Now you can construct a "narrow" distribution $Y_n \ni$

$$(*) \qquad E(Y_n^j \mid X_1) = E(Y^j \mid X_1) \quad j = 1, \ldots, 2k-1$$

with

$$(**) \qquad E(Y_n^{2k} \mid X_1) \leq E(Y^{2k} \mid X_1)$$

If you know the "endpoints of X_1" you can construct a "broad" dist Y_b where $(*)$ holds with Y_n replaced by Y_b and

$$(***) \qquad E(Y^{2k} \mid X_1) \leq E(Y_b^{2k} \mid X_1).$$

<u>Remark</u> $(**)$ + $(***)$ seem to be analogous to putting bounds on cells in tabular data.