Heterogeneous beliefs, model uncertainty and asset pricing

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Do Heterogeneous Beliefs Matter for Asset Pricing?

The Review of Financial Studies, 2005

The Impact of Risk and Uncertainty on Expected Returns

Representative agent models can not explain asset prices

Equity premium puzzle (Mehra and Prescott 1985).

- Average real rate of return on stocks is 7% per year
- Average real rate of return on bonds is 1% per year
- In order to account for the equity premium agents have to be extremely risk-averse in standard representative agent models

Many different approaches have been taken to account for the equity premium puzzle.

Two recent approaches that have received attention are

- Bansal and Yaron (2004)
- Campbell and Cochrane (1999)

Heterogeneity

Economic agents differ in their preferences, endowments and beliefs.

We ignore differences in preferences, allow for different endowments, and focus on different beliefs.

The heterogeneity of beliefs captures how individual agents

1. interpret the same information differently, and/or
2. have access to differing information sets.

- We propose to measure the beliefs of agents with the beliefs of professional forecasters (e.g. stock market analysts)
- Since in reality the beliefs of agents are different from the beliefs of forecasters, we allow for biases in some specifications.

Some prior related work

We are not the first to study asset pricing models with heterogeneous agents.

Most papers discuss heterogeneous agents in the context of incomplete market models.

- An important example is Duffie and Constantinides (1996).

The closest paper in the literature to our work is Shefrin (2001).

Using the dispersion of analysts' one-year earnings forecasts, Diether, Malloy, and Scherbina (2002) find that higher dispersion stocks have lower future returns.

What do we do differently?

- Explore analyst data for the purpose of empirical asset pricing and link the data with SDF asset pricing models.
- In early work, Ghysels and Juergens (2001), and more recent work by Anderson, Ghysels, and Juergens (2004) and Qu, Starks, and Yan (2003) find that factors for dispersion of beliefs are priced in traditional factor asset pricing models.
- We estimate Euler equations with heterogeneous beliefs, where beliefs are taken from analysts.
- We consider both
  - Heterogeneous agent interpretation.
  - Single-agent model uncertainty interpretation.

Structure of Talk

1. Asset pricing with heterogeneous beliefs

2. Empirics H

3. Asset pricing with model uncertainty

4. Empirics U

A model with heterogeneous agents

- Complete markets with I types of agents and the lifetime utility function for agents of type i is

$$E_{i0}\sum_{t=0}^{T}\beta^t\frac{{c_{it}}^{1-\gamma}}{1-\gamma}.$$

- The budget constraint of a type i agent at time $t<T$ is $c_{it}+\sum_{p=1}^{n}\varphi_{ipt}=w_{it}+\sum_{p=1}^{n}\varphi_{ipt-1}r_{pt}$ and at time T, $c_{iT}=w_{iT}+\sum_{p=1}^{n}\varphi_{ipT-1}r_{pT}$

- The beliefs of any agent are absolutely continuous with respect to the true probabilities and vice versa.

- For each agent i : $1=E_{it}\left[\beta\left(\frac{c_{it}}{c_{it+1}}\right)^{\gamma}r_{pt+1}\right].$

Forecasters' beliefs as proxies for agents' beliefs

Version one

- Beliefs of forecasters are the beliefs of agents

Version two: amplification

- Mean beliefs of forecasters are the mean beliefs of agents
- Dispersion of forecasters' beliefs is amplified to obtain the dispersion of agents' beliefs

Version three: biases and amplification

- Systematic differences between the mean beliefs of forecasters and agents.
- Dispersion of forecasters' beliefs is amplified to obtain the dispersion of agents' beliefs

Asset pricing

For each agent i

$$1 = E_{it}\left[\beta\left(\frac{c_{it}}{c_{it+1}}\right)^{\gamma} r_{pt+1}\right].$$

Under additional assumptions the pricing equation will become:

$$\begin{aligned} 1 &= \textstyle\sum_{i=1}^{I} \lambda_{it} E_{it}\left[\beta g_{t+1}^{-\gamma} r_{pt+1}\right] \\ &= \beta h_{pt} E_t g_{t+1}^{-\gamma} r_{pt+1} \end{aligned}$$

where $g_{t+1} = c_{t+1}/c_t$ is the gross rate of aggregate consumption growth and

$$h_{pt} = \left[\sum_{j=1}^{I} \lambda_{jt}\left(\frac{\mu_{gjt+1|t}}{\mu_{gt+1|t}}\right)^{-\gamma}\left(\frac{\mu_{pjt+1|t}}{\mu_{pt+1|t}}\right)\right]$$

Assumptions

- Constant distribution across agents
  - A certain function of Pareto weights is constant
- Mean beliefs
  - Stock returns
  - Consumption growth
- The distribution of each agents' beliefs
  - Homotopy
  - Limited rational expectations

Constant distribution

Let λ_{it} be the time t Pareto weight for agents of type i.

To compute an equilibrium we can solve the following Pareto optimal problem: maximize

$$\sum_{s=t}^{T}\sum_{i=1}^{I}\beta^{s-t}\lambda_{it}E_{it}\frac{{c_{is}}^{1-\gamma}}{1-\gamma}$$

subject to $\sum_{i=1}^{I} c_{is} \leq c_s$ and $c_{is} \geq 0$. for all dates and histories.

Assumption: The distribution of agents, as measured by, $\sum_{i=1}^{I}(\lambda_{it})^{\frac{1}{\gamma}}$, is constant over time.

Implication: The pricing equation can be written as

$$1 = \sum_{i=1}^{I}\lambda_{it}E_{it}\left[\beta\left(\frac{c_t}{c_{t+1}}\right)^{\gamma} r_{pt+1}\right].$$

Measuring beliefs

Forecasters' beliefs as proxies for agents' beliefs

We are limited by the available data:

- First Call/IBES: Earnings
- Survey of professional forecasters: Corporate profits

Aggregation: We aggregate all of the financial analysts in a brokerage house to obtain one fictional forecaster.

The number of types of agents, I, is equal to the number of brokerage houses.

The Pareto weight of agents of type i is determined by the rank of analysts from brokerage house i.

Notation

At time t for a variable x_{t+1}, let

- the true conditional expectation be $\mu_{xt+1|t}$
- type i agents' conditional expectation be $\mu_{xit+1|t}$
- analyst i's conditional expectation be $\bar{\mu}_{xit+1|t}$

In version one, we set

$$\mu_{git+1|t} = \bar{\mu}_{git+1|t}$$
$$\mu_{pit+1|t} = \bar{\mu}_{pit+1|t}$$

Earnings forecasts → return forecasts

Assumption: At time t, forecaster i's belief about the expected return on stock p between periods t and $t+1$ is

$$\bar{\mu}_{pit+1|t} = 1 + \frac{\Delta_{ipt}}{Q_{pt}} + \delta_{ipt}$$

where for stock p and forecaster i :

- Δ_{ipt} is a short term forecast of net income per share.
- Q_{pt} is the current-month price, and
- δ_{ipt} is a long term earnings growth forecast.

Problem: Net income per share may be serially correlated.

- This may lead us to understate the dispersion in returns.
- Later on, we will present one possible solution to this problem.

Beliefs about consumption growth

Assumption: Agent i's belief about the conditional mean of aggregate consumption growth, $\mu_{git+1|t}$, satisfies

$$\log \mu_{git+1|t} = q_t + \phi \log \mu_{mit+1|t}$$

where

- ϕ is a constant
- q_t is an unknown function of current information
- $\mu_{mit+1|t}$ is agent i's forecast of the market return.

Homotopy

Assumption: At time t, for a positive variable x, let the true conditional cdf of x_{t+1} be $\xi_{xt}(x_{t+1})$. The conditional cdf of x_{t+1} given time t information from the point of view of agent i is

$$\xi_{ixt}(x_{t+1}) = \xi_{xt}\left(x_{t+1} * \mu_{xt+1|t} \; / \; \mu_{xit+1|t}\right).$$

Under homotopy

- If an agent is correct about the conditional mean then he is correct about the entire distribution.
- If agent i is more optimistic about the conditional mean than agent j then agent i is globally more optimistic than agent j
- All agents correctly know the second and higher order conditional central moments of $\log(x)$.

Limited rational expectations

Analysts' expectations of consumption growth and stock returns are correct on average:

$$\mu_{gt+1|t} = \sum_{i=1}^{I} \lambda_{it}\mu_{git+1|t}$$
$$\mu_{pt+1|t} = \sum_{i=1}^{I} \lambda_{it}\mu_{pit+1|t}.$$

Agents have limited rational expectations

- rational expectations about some variables
- irrational expectations about others variables

Asset pricing

The pricing equation becomes:

$$\begin{aligned} 1 &= \sum_{i=1}^{I} \lambda_{it} E_{it} \left[\beta g_{t+1}^{-\gamma} r_{pt+1} \right] \\ &= \beta h_{pt} E_t g_{t+1}^{-\gamma} r_{pt+1} \end{aligned}$$

where $g_{t+1} = c_{t+1}/c_t$ is the gross rate of aggregate consumption growth and

$$h_{pt} = \left[\sum_{j=1}^{I} \lambda_{jt} \left(\frac{\mu_{gjt+1|t}}{\mu_{gt+1|t}} \right)^{-\gamma} \left(\frac{\mu_{pjt+1|t}}{\mu_{pt+1|t}} \right) \right]$$

Summary of the assumptions

- Constant distribution across agents
  - A certain function of Pareto weights is constant
- Mean beliefs
  - Stock returns
  - Consumption growth
- The distribution of each agents' beliefs
  - Homotopy
  - Limited rational expectations

Empirical results - Heterogeneous agent models

Some simple things - I

Summary Statistics of Fama-French, Momentum, and Dispersion Factors

				ACF		
	Mean	**Std. Dev.**	**t-stat**	**1**	**2**	**12**
$\mathbf{r_m - r_b}$	1.16	3.09	3.46	-0.21	0.11	-0.06
SMB	0.14	2.68	0.47	0.10	-0.09	0.10
HML	0.47	2.44	1.75	0.21	-0.06	0.13
UMD	0.78	2.52	2.84	0.04	0.01	0.12
DISP	-0.28	1.44	-1.75	0.14	0.00	0.16
LTGDISP	0.23	0.27	0.96	0.09	-0.07	0.11

Some simple things - II

Regressions of S&P 500 Excess Returns on FF, Momentum and DISP Factors

Intercept	$\mathbf{r_m - r_b}$	SMB	HML	UMD	DISP	LTGDISP
0.01	1.10	0.08	0.14	-0.13		
(0.14)	(47.98)	(3.10)	(4.60)	(-4.94)		
0.15	1.02				0.26	
(1.99)	(45.92)				(5.39)	
0.06	1.01					0.12
(0.74)	(40.68)					(3.36)
0.06	1.08	0.05	0.11	-0.10	0.12	
(0.86)	(45.83)	(1.94)	(3.54)	(-3.92)	(2.37)	
-0.01	1.08	0.05	0.15	-0.11		0.09
(-0.09)	(48.07)	(1.69)	(5.12)	(-4.47)		(2.79)

GMM estimation

A fixed weighting matrix with five assets:

- The market
- A nominal risk-free bond
- A portfolio with a high degree of past volatility
- A portfolio with a high degree of long term dispersion
- A portfolio with a high degree of short term dispersion

When beliefs coincide with analysts not much happens

Recall the pricing equation

$$1 = \beta h_{pt} E_t g_{t+1}^{-\gamma} r_{pt+1}$$

where

$$h_{pt} = \left[\sum_{j=1}^{I} \lambda_{jt} \left(\frac{\mu_{gjt+1|t}}{\mu_{gt+1|t}} \right)^{-\gamma} \left(\frac{\mu_{pjt+1|t}}{\mu_{pt+1|t}} \right) \right].$$

h_{pt} is close to one because forecasts are similar.

The maximum monthly different in monthly forecasts of the market (high-low)

- 0.15% for the market
- 5% for the high short term dispersion portfolio

Version 2: Amplification

In model two, we amplify the differences of analysts' forecasts, as the diversity of analysts' forecasts may not fully represent the diversity of agents' beliefs.

We introduce a parameter θ that can amplify or dampen heterogeneity by amplifying or dampening the beliefs of financial analysts.

The ratio of beliefs of type i agents of the conditional means of consumption growth and stock returns to the true conditional means is

$$\frac{\mu_{xit+1|t}}{\mu_{xt+1|t}} = \frac{\left(\bar{\mu}_{xit+1|t}\right)^{\theta}}{\sum_{k=1}^{I} \lambda_{kt} \left(\bar{\mu}_{xkt+1|t}\right)^{\theta}}$$

Estimating θ provides a convenient way to test for the importance of dispersion.

Estimates with amplification

Cons. resp.	Ampl.			
ϕ	θ	β	γ	**Objective**
1	0	0.966*	457.29*	0.3692
1	1	0.953*	459.70*	0.3618
0.05	1	0.966*	457.31*	0.3690
1	−4.60*	0.688*	497.00*	0.2932
1	3.68*	0.999	229.87*	0.3188
0.05	19.01*	0.956*	453.64*	0.3104
0.05	18.93*	0.999	439.54*	0.3110

Biases

Model assumes that beliefs of analysts and agents differ and we use data on the buy/sell recommendations of analysts to approximate the difference of the beliefs of analysts and agents.

Financial analyst i's recommendation for stock j is represented as $\mathcal{R}_{ij} = -2, -1, 0, 1, 2$ if *strong sell, sell, hold, buy, strong buy,* respectively.

A measure for the bias in portfolio p (with stock j having share ϑ_j) at time t as

$$b_{pt+1|t} = \sum_{i=1}^{I} \sum_{j=1}^{m} \lambda_i \vartheta_j \mathcal{R}_{ij}$$

We do not permit biases in all variables. We assume

- no biases about the market
- no biases about nominal risk-free bonds
- no biases about consumption growth

The language of analysts

It is well known that analysts on average recommend buying stocks. In our sample, the average recommendation across all analysts of the market is 0.8118.

We adjust analysts recommendations of all stocks by the time-invariant constant $\zeta = 0.8118$. From now on

- when we say a financial analyst recommends holding a stock we mean that the recommendation for the portfolio is ζ.
- Any recommendation above (below) ζ signifies a buy (sell) recommendation.
- The difference between the recommendation and ζ measures the strength of the buy (sell) recommendation.

Incorporating bias

Assumption: The ratio of beliefs of type i agents about the conditional mean of consumption growth and the conditional mean of the return on stock p to the true conditional means are:

$$\frac{\mu_{git+1|t}}{\mu_{gt+1|t}} = \frac{\left(\bar{\mu}_{git+1|t}\right)^{\theta}}{\sum_{k=1}^{I} \lambda_{kt} \left(\bar{\mu}_{gkt+1|t}\right)^{\theta}}$$

$$\frac{\mu_{pit+1|t}}{\mu_{pt+1|t}} = \left[\frac{\left(\bar{\mu}_{pit+1|t}\right)^{\theta}}{\sum_{k=1}^{I} \lambda_{kt} \left(\bar{\mu}_{pkt+1|t}\right)^{\theta}}\right] \exp\left[-d\left(b_{pt+1|t} - \zeta\right)\right]$$

Strong recommendations signify that analysts view the stock more favorably than agents.

Biased expectations for agents not analysts

We continue to maintain the assumption that financial analysts are correct on average about actual expected returns.

Agents are correct on average about consumption growth but possibly incorrect on average about some assets when $d \neq 0$ since

$$\sum_{i=1}^{I} \lambda_{it} \frac{\mu_{pit+1|t}}{\mu_{pt+1|t}} = \exp\left[-d\left(b_{pt+1|t} - \zeta\right)\right].$$

Estimates allowing for bias

Cons. resp.	Bias	Ampl.			
ϕ	d	θ	β	γ	**Objective**
1	0	0	0.966*	457.29*	0.3692
1	0	1	0.953*	459.70*	0.3618
1	0	−4.60*	0.688*	497.00*	0.2932
1	0	3.68*	0.999	229.87*	0.3188
0.05	0	1	0.966*	457.31*	0.3690
0.05	0	19.01*	0.956*	453.64*	0.3104
0.05	0	18.93*	0.999	439.54*	0.3110
1	0.048*	0	0.879*	498.59*	0.2501
1	0.047*	−4.51*	0.674*	509.04*	0.1849
0.05	0.044*	17.91*	0.928*	473.54*	0.2120
1	0	0	1.005*	5	0.6038
1	0.048*	43.90*	1.003*	5	0.4038
0.05	0.059*	243.60*	0.991*	5	0.4014

Note: $d = 0$, no bias, $\theta = 1$, no amplification.

Plausible estimates?

When we include biases we find that optimal estimates of d are remarkably stable and range between 0.04 and 0.06.

If $d = 0.05$ and $\zeta = 0.8118$ then when analysts on average

- state a buy recommendation, analysts are about 1% more optimistic than individuals about returns on the asset since $\exp\left[-d\left(1-\zeta\right)\right] \approx .99$.
- state a hold recommendation, analysts are about 4% more pessimistic than individuals about returns on the asset since $\exp\left[-d\left(0-\zeta\right)\right] \approx 1.04$.

Many of our estimates of θ also seem plausible.

Core uncertainty

Consider:

$$E_t r_{t+1} = \mu + \gamma \textbf{var}_t(r_{t+1}) + \theta \textbf{unc}_t(r_{t+1}).$$

We measure $\textbf{unc}_t(r_{t+1})$ with the disagreement of forecasters on future corporate profits. We consider three versions:

- $\theta = 0$.
- $\textbf{unc}_t$ is given by the variance of corporate profits growth forecasts.
- $\textbf{unc}_t$ is given by a beta weighted variance of corporate profits growth forecasts.

Risk-Return Relationship

- Expected excess market returns vary positively and proportionally to conditional volatility of the market return
  - Mixed empirical evidence
    - No significant relation: Baillie and DeGennaro (1990), Campbell and Hentschel (1992) French, Schwert, and Stambaugh (1987)
    - Significantly negative: Brandt and Kang (2004), Campbell (1987), Nelson (1991)
    - Mixed positive/negative: Glosten, Jagannathan, and Runkle (1993), Harvey (2001), Turner, Startz, and Nelson (1989)
    - Significantly positive: Ghysels, Santa-Clara, and Valkanov (2005)

Empirical Measure of Model Uncertainty

- Existing literature measures disagreement with flat-weighted variances
  - No relation to expected returns
- Instead use a flexible weighting scheme inspired by MIDAS regressions: Ghysels, Santa-Clara, and Valkanov (2005)
- unc_t is time series process measuring model uncertainty from data on aggregate corporate profits by professional forecasters. Observe that disagreement only matters when the extreme forecasts are ignored

The Reference Model and Uncertainty

- General equilibrium model where all agents are alike and have power utility functions

$$dx_t = (a_t - \Delta_t g_t)dt + \Lambda_t dB_t$$
$$dP_t = (\lambda_t - \eta_t g_t)P_t dt + \zeta_t P_t dB_t$$
$$dP_t = (\psi_t \lambda_t y_t - \psi_t \lambda_t y_t \eta_t g_t + \rho_t y_t - c_t)dt + \psi_t \zeta_t y_t dB_t$$

- Agents believe the reference model is a reasonable approximation but they worry the approximation may be misspecified
- Setup follows Merton (1973), Hansen and Sargent (2001), Uppal and Wang (2003), and Maenhout (2004)

The Objective Function

- The objective of agents is taken to be

$$\int_0^\infty \exp(-\delta t)\left[\frac{c_t^{1-\gamma}}{1-\gamma}+\frac{1}{2\theta}g_t' g_t\right]$$

- where the first component is the utility obtained from consumption and the second component penalizes deviations from the reference model
- The minimizing choice of $g = \theta\Delta' J_x + \theta\eta' J_y \psi y$
- where J_x, J_y are derivatives of the Hamilton-Jacobi value function and θ, Δ, and η endogenously determine the perturbations of the conditional means

Discrete Time Approximation

- In our empirical work, we apply a discretization of the above model in which quarterly market excess returns satisfies:

$$E_t r_{et+1} = \gamma V_t + \theta M_t + H_t \text{ , where}$$

$$V_t = \zeta_t \zeta_t'$$

$$M_t = \eta_t \eta_t'$$

$$H_t = \theta \overline{\eta}_t \overline{\Delta}_t' \frac{J_x(x_t, y_t)}{J(x_t, y_t)} - tr\left[\zeta_t \Lambda_t' \frac{J_{xy}(x_t, y_t)}{J_y(x_t, y_t)}\right]$$

Survey of Professional Forecasters (SPF)

- We utilize the SPF to represent the beliefs of agents, where dispersion about variables proxies for the amount of model uncertainty agents have about the reference model.
- Advantages:
  - Long time series (4Q1968 – 3Q2003)
  - Multiple horizon predictions
  - Better than most survey data
  - Forecasts of output, output deflator, and corporate profits after taxes

Constructed Variables

- Approximate growth rates of GDP and corporate profits

$$\left(\frac{E_{it} X_n E_{it} P_m}{E_{it} X_m E_{it} P_n} \right)^{\frac{1}{n-m}}$$

- Real market return
  - Gordon growth model

$$E_{it} r_{mt+1} = E_{it} \left[\frac{\pi_{t+1}}{q_t} \right] + \xi_{it}$$

- Real return on risk-free bond

$$E_{it} r_{bt+1} = \frac{R_{bt+1} P_t}{E_{it} P_{t+1}}$$

Empirical Estimation

- Recall: $E_t(r_{et+1}) = b + \gamma V_t + \theta M_t$
- where H_t is now equal to 0
  - Noise in market uncorrelated to noise in state
  - Noise in market uncorrelated with uncertainty in state
- Risk-return trade-off
  - Measure using Ghysels et al. MIDAS approach
- Uncertainty-return trade-off
  - Summarize cross-sectional variation with parametric specification
- Estimate via QMLE because model may be misspecified

Volatility

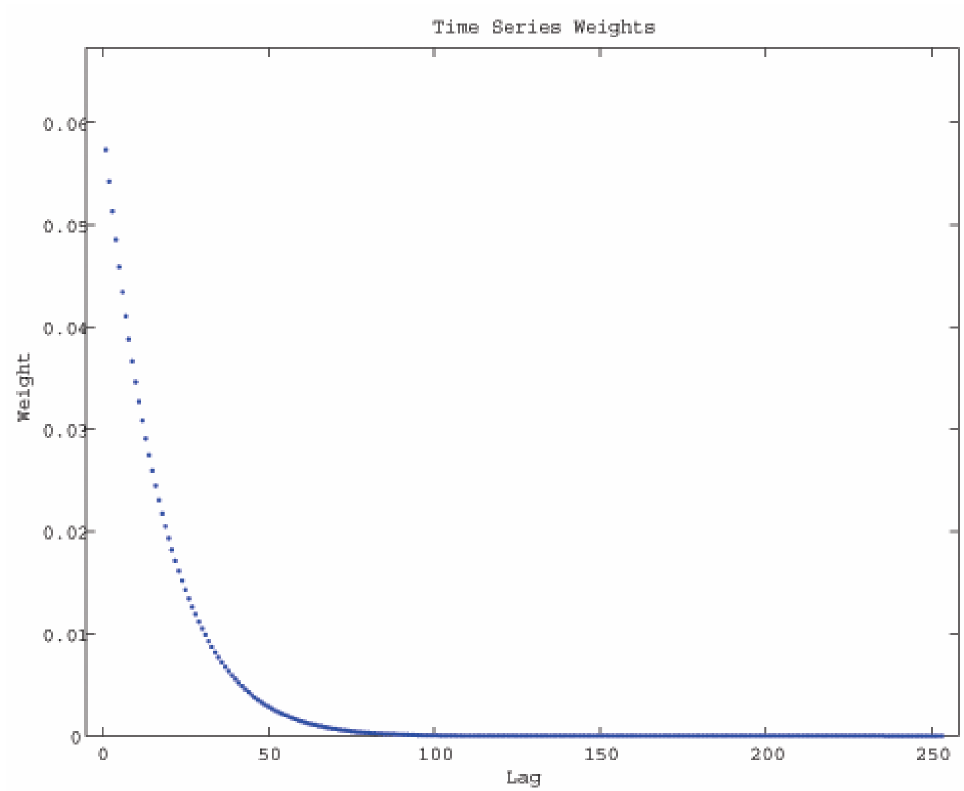
- Conditional volatility:

$$V_t = \sigma^2 vol_t(\omega)$$

- where ω comes from

$$l_i(\omega) = \frac{(s-i)^{\omega-1}}{\sum_{j=1}^{s} (s-j)^{\omega-1}}$$

- a MIDAS weighting scheme with single parameter controlling a lag polynomial



Model Uncertainty

- The contribution of model uncertainty to excess returns is captured by θunc_t
  - θ is a time invariant constant and unc_t measures disagreement about the growth rate of a single variable
    - Market return forecasts, constructed GDP and corporate profit growth forecasts
    - Now, we use the MIDAS estimation to capture weights in the cross-section

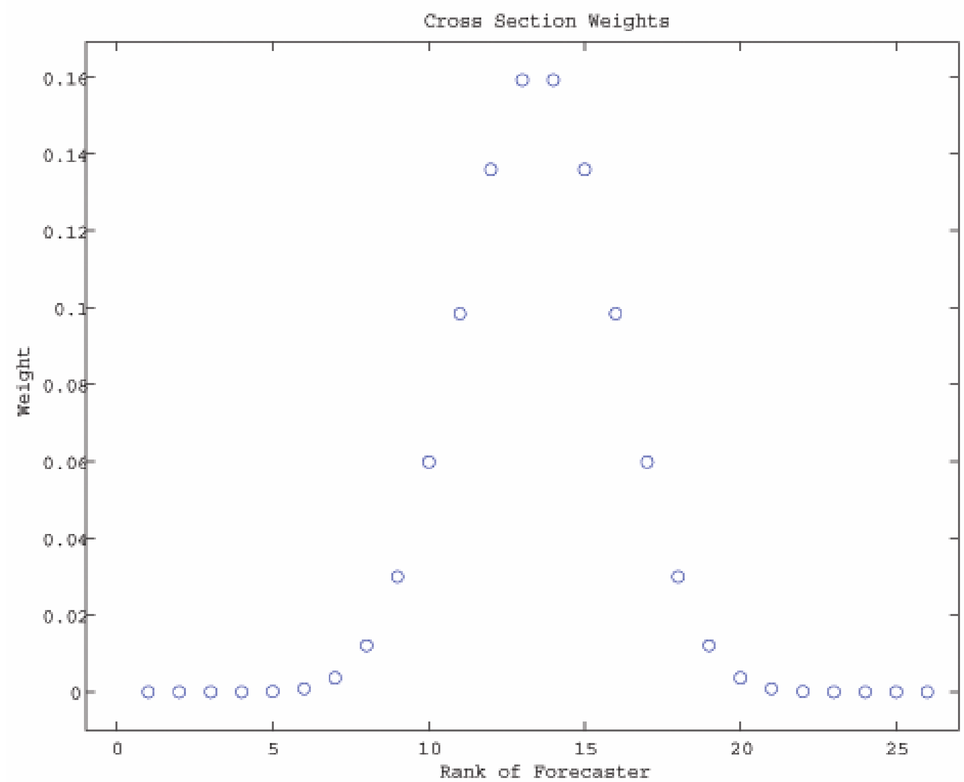
Model Uncertainty

- Conditional volatility:

$$\text{unc}_t(\nu) = \sum_{i=1}^{f_t} w_{it}(\nu)\left[x_{it+1|t} - \sum_{j=1}^{f_t} w_{jt}(\nu) x_{jt+1|t}\right]^2$$

- Both power parameters are set equal to each other and are estimated by a single common parameter, ν

$$w_{it}(\nu) = \frac{i^{\nu-1}(f_t - i)^{\nu-1}}{\sum_{j=1}^{f_t}(f_t - j)^{\nu-1}}$$



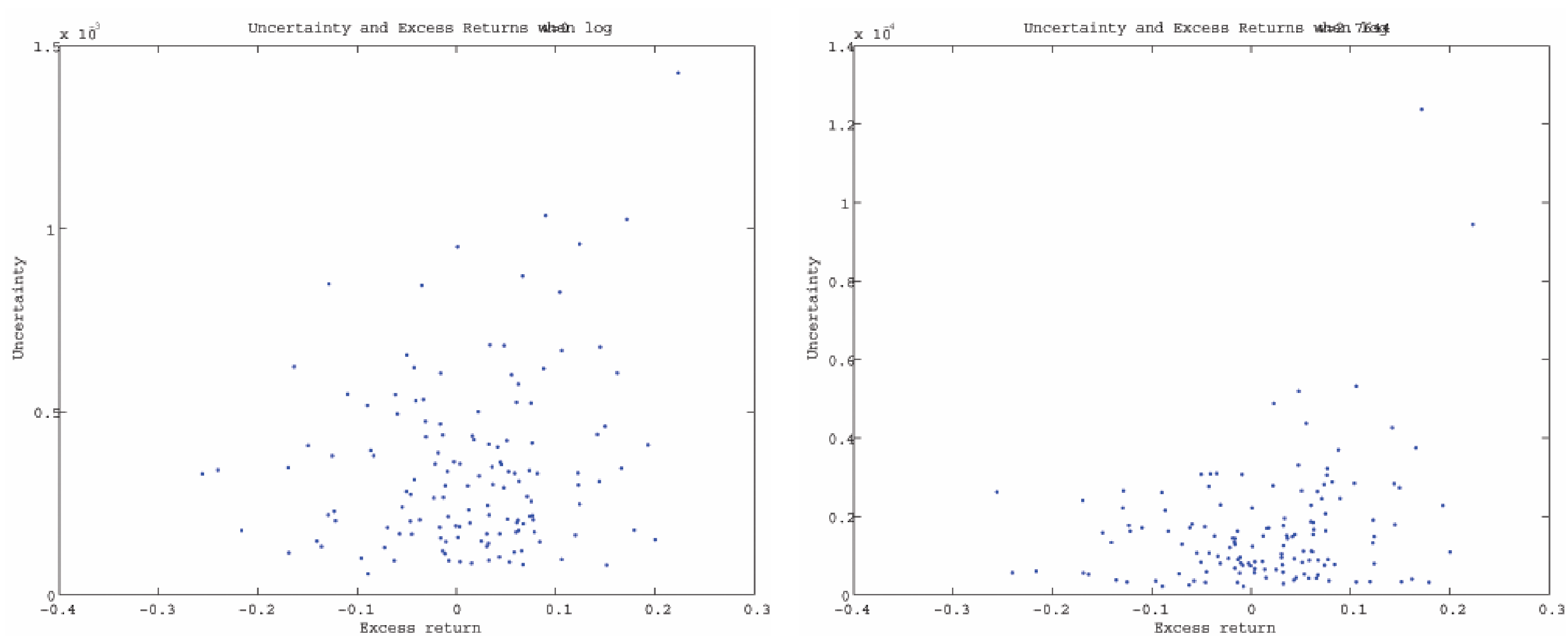
- We find stronger empirical evidence for a model uncertainty-return trade-off than for a traditional risk-return trade-off
  - Slight evidence of pessimism

Table 3: Properties of Uncertainty and Volatility

Correlations of Excess Returns with vol and unc

	r_{et+1}	$RVol_{t+1}$	$RVol_t$	$vol_t(\hat{\omega})$	$unc_t(1)$	$unc_t(\hat{\nu})$
r_{et+1}	1.000	-0.397	0.128	0.154	0.175	0.283
$RVol_{t+1}$		1.000	0.202	0.312	0.051	0.004
$RVol_t$			1.000	0.748	0.145	0.081
$vol_t(\hat{\omega})$				1.000	0.211	0.075
$unc_t(1)$					1.000	0.662
$unc_t(\hat{\nu})$						1.000

Figure 2: Uncertainty and Returns



Conclusion

- In recent years we have moved away from the rational expectations representative agent paradigm of market efficiency.
- New models have agents with different beliefs, agents unsure about the ‘true’ model
- Empirical work shows promise!