Introduction:

Credit Risk: The distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement

Probability of Default: Any type of failure to honor a financial agreement

Structural Models vs Reduced Models

Two competing models that exist in this area. Jarrow and Protter (2004) present a comparison stating:

“ These models are not disconnected and disjoint model types as is commonly supposed, but rather they are really the same model containing different information assumptions ”

Structural Model: Assumes certain knowledge of a very detailed information set (akin to that held by firm managers)
Most cases assume default time is predicatable (except in the case where a firms asset value follows a continuous time jump diffusion process)
Reduced Model: Assumes knowledge of a less detailed information set (akin to that observed by the market)
Most cases assumes default time as inaccessible

Difference: Not in characterization of default time *(predictable vs. inaccessible)* but in the information set available to the modeler.

Structural Model can be transformed to reduced form as the information set changes and is less refined. A consequence of this is that the current debate in Credit Risk literature is misdirected

DEBATE SHOULD NOT BE:

Which model is best in terms of forecasting performance

DEBATE SHOULD BE:

Focused on whether model should be based on the information set observed by the market or not

Structural Credit Models

Basis: Corporate liabilities are contingent claims on the assets of a firm. The market value of the firm is the fundamental source of uncertainty driving credit risk.

Classical Approach

Market Value: X = expected discounted future cash flows of the firm Firm is financed by equity and a zero coupon bond with face value K and maturity date T. If the firm cannot fufill its payment obligation, then bond holders will immediately take over the firm, hence default time is given by

$$\tau = \begin{cases} T \text{ if } V_T < K \\ \infty \text{ otherwise} \end{cases}$$

Standard model for the firm's asset value:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t \ , X_0 > 0$$

where μ is a drift paramter, $\sigma > 0$ is a volatility parameter, and W is a standard Brownian motion,
Since $W_T \tilde{N}(0, T)$, default proabababilities $p(T)$ can be shown as:

$$p(T) = Pr[X_T < K] = Pr[\sigma W_T < logL - mT] = \Phi\big(\frac{logL - mT}{\sigma\sqrt{T}}\big)$$

where $L = \frac{K}{X_0}$ is the initial leverage ratio and $m = \mu - \frac{1}{2}\sigma^2$

PAYOFFS AT MATURITY, T, IN THE CLASSICAL APPROACH

If $X_T \geq K$ *no default* bondholders get K and shareholders get $X_T - K$

If $X_T < K$ *default* then ownership of firm goes to bondholders who lose $K - X_T$

AND Equity is worthless b/c of limited liability

To summarize this, the value of the Bond issue B_T^T at time T is given by

$$B_T^T = min(K, X_T) = K - max(0, K - X_T)$$

This payoff is equivalent to that of a portfolio composed of a default-free loan with face value K maturing at T and a short European put position on the assets of the firm with strike K and maturity T.

The value of the equity E_T at time T can be shown as:

$$E_T = max(0, X_T - K)$$

This is equivalent to a European call option on the assets of the firm with strike K and maturity T.

PRICING EQUITY AND CREDIT RISKY DEBT REDUCES TO PRICING EUROPEAN OPTIONS

Credit Spread is the difference between the yield on a defaultable bond and the yield on an otherwise equivalent default-free zero bond. It gives the excess return demanded by bond investors to bear the potential default losses.
Since the yield $y(t,T)$ on a bond with price $b(t,T)$ satisfies $b(t,T) = e^{-y(t,T)(T-t)}$ the credit spread,$S(t,T)$ at time t is given by,

$$S(t,T) = -\frac{1}{T-t} log\big(\frac{B_t^T}{\overline{B}_t^T}\big), \quad T > t$$

where $\overline{B}_t^T$ is the price of a default free bond maturing at T. The term structure of credit spreads is the schedule of $S(T,t)$ against T

.

FIRST PASSAGE APPROACH:

In the classical approach, the firm value can dwindle to almost nothing without triggering default which is an unfavorable occurrence for bond holders. Often safety covenants give bond investors the right to reorganize a firm if its value falls below a given barrier, D.

Suppose the default barrier D is a constant, valued in $(0, X_0)$. The default time τ is a continuous random variable valued in $(0, \infty)$ given by

$$\tau = inf(t > 0 : X_t < D)$$

In this first passage approach, the default time is calculated as any point that the asset value falls below the default value D, which may or may not be before time to maturity T.

Proabability to default can now been written as

$$p(T) = Pr[M_T < D] = Pr[min_{s \leq T}(ms + \sigma W_s) < log(\frac{D}{X_0})]$$

since the distribution of the historical low of an arithmetic Brownian motion is inverse Gaussian it can be shown as

$$\phi(log\frac{\frac{D}{X_0} + mT}{\sigma\sqrt{T}})[1 + (\frac{D}{X_0})^{\frac{2m}{\sigma^2}}]$$

M is the historical low of firm values with $M_t = min_{s \leq t} X_s$.

In Summary there are Two Scenarios to consider

1) $D \geq K$. *the Barrier is greater than the face value*

If the firm value never falls below the barrier D over the term of the bond ($M_T > D$), then the bond investors receive the face value $K < X_0$ and the equity holders get the remaining $X_T - K$

However, if the firm falls below the barrier at some point during the bond's term ($M_T \leq D$) then the firm defaults. In this case the firm stops operating, bond investors take over its assets D and equity investors receive nothing.

Bond investors are fully protected and they recieve at least the face value K upon default, the bond is not subject to default risk any more.

2)When $D < K$ *the Face value is greater than the barrier* such that bondholders are exposed to some default risk and likewise compensated for bearing that risk

If $M_T > D$ and $X_T \geq K$ then bond investors receive the face value K and the equity holders receive the remaining $X_T - K$.

If $M_T > D$ but $X_T < K$ then the firm defaults, since the remaining assets are not sufficient to pay off the debt in full, and bond investos collect the remaining assets X_T and equity becomes worthless.

If $M_T \leq D$ then the firm defaults as well. Bond investors receive $D < K$ at default and equity becomes worthless.

Reisz and Perlich (2004) point out that if the barrier is below the bond's face value, then the earlier definintion of τ does not reflect economic reality anymore
it does not capture the situation when the firm is in default because $X_T < K$ although $M_T > D$.

From the following situations it is now necessary to redefine default

REDEFINING DEFAULT

We have shown that default can occur at different instances and therefore it is appropriate to redefine the default τ

We redefine default as firm value falling below the barrier $D < K$ at any time before maturity

OR

Firm value falling below face value K at maturity.

$$\tau = min(\tau^1, \tau^2)$$

where τ^1 is the first passage time of assets to the barrier D and τ^2 is the maturity time T if assets $X_T < K$ at time T and ∞ otherwise.

Default proability can now be shown as:

$$p(T) = 1 - (r[min(\tau^1, \tau^2) > T] = 1 - Pr[M_T > D, X_T > K])$$

$$= 1 - Pr[min(t \leq T)(mt + \sigma W_t) > log(\frac{D}{X_0}), mT + \sigma W_T > logL]$$

through the joint distribution of an arithmetic Brownian and its running minimum we get

$$p(T) = \phi(\frac{logL - mT}{\sigma\sqrt{T}}) + (\frac{D}{X_0})^{\frac{2m}{\sigma^2}} \phi(\frac{log(\frac{D^2}{KX_0} + mT}{\sigma\sqrt{T}})$$

Corresponding Payoff to equity

$$E_T = max(0, X_T - K)1_{M_T \geq D}$$

This equity position is equivalent to a European down and out call option position on firm assets X with strike K and barrier $D < K$, and maturity T

Corresponding Payoff to bond investors at maturity

$$B_T^T = K - (K - X_T)^+ + (X_T - K)^+ 1_{M_T < D}$$

This position is equivalent to a portfolio composed of a risk free loan with face value K maturing at T, a short European put on the firm with strike K and maturity T and a long European down-and-in call on the firm with strike K and maturity T.

IN the first passage approach - bonds are worth at least as much as in the classical approach.
- bond investors have aditionally a barrier option on the firm that knocks in if the firm defaults before the maturity T.
Correspondingly

$$B_0^T = X_0 - C(\sigma, T, K, X_0) + X_0 \Big(\frac{D}{X_0}\Big)^{\frac{2r}{\sigma^2}+1} \phi(h_+) + K e^{-rT} \Big(\frac{D}{X_0}\Big)^{\frac{2r}{\sigma^2}-1} \phi(h_-)$$

which again implies the value identity $X_0 = S_0 + b_0^T$

It is important to make note that: with increasing T, the credit spread approaches zero asymptotically, which is at odds with the empirical observation that credit spreads tend to increase with increasing maturity, reflecting the fact that uncertainty is greater in the distant future than in the near term.

TIME VARYING DEFAULT BARRIER

To avoid this inconsistency we introduce a time varying default barrier $D(t) \leq K$ for all $t \leq T$,

For some constant $k > 0$ consider the deterministic function $D(t) = Ke^{-k(T-t)}$ which can be thought of as the face value of the debt, discounted back to time t at a continuously compounding rate k. The firm defaults at $\tau = inf\{\tau > 0 : X_t < D(t)\}$

Observing that

$$\{X_t < D(t)\} = \{(m-k)t + \sigma W_t < logL - kT\}$$

we have the default probability

$p(T) = Pr[min_{t \leq T}((m-k)t + \sigma W_t) < logL - kT]$

which can be seen as

$$p(T) = \phi(\frac{logL - mT}{\sigma\sqrt{T}}) + \left(Le^{-kT}\right)^{\frac{2m}{\sigma^2}(m-k)} \phi(\frac{log(L) + (m-2k)T}{\sigma\sqrt{T}})$$

with $(m-k)$ as the drift

The corresponding Equity position, which is a European down-and-out call option with strike K, time varying paramter $D(t)$, and maturity T as $E_T = (X_T - K)^+ 1_{m_T^k \geq D}$
M_t^k is the minimum of the firm value with adjusted arithmetic growth rate (m-k) and $D(t) = Ke^{-kT}$
The bond position is given by
$B_T^T = K - (K - X_T)^+ + (X_T - K)^+ 1_{M_T^k < D}$

REDUCED FORM MODELS

First introduced by Jarrow and Turnbull (1992) KEY : the modeler observes the filtration generated by a random default time τ The default time is a stopping time generated by a Cox process $N_t = 1_{r \leq t}$ which is a point process with one jump of size one at default. This default process is increasing and thus has an upward trend.

Using the Doob-Meyer decomposition theorem: There exists an increasing process A^τ starting at zero such that $N - A^\tau$ becomes a martingale. The unique process A^τ counteracts the upward trend in N and is thus called the compoensator which is continuous iff τ is unpredictable. We can model this compensator through a non-negative process λ. Since any given non-negative process can be used to parameterize the dynamics of default, no economic model of firm default is needed for this purpose any more!

The defining characteristics of the models is not the property of the default time but rather the information structure of the model itself. INTEREST IN: pricing a firm's risky debt or related credit derivatives, reduced form models are preferred approach. Reduced form models have been constructed purposefully to be based on the information avaiable to the market.

General Framework

- Consider a reduced form credit risk model for firm's risky debt.
- If firm defaults prior to time T, a *recovery rate* between 0 and 1 will be paid per promised dollar.
- Traditional reduced form approach assumes default is absorbing state.
- Guo, Jarrow, and Zeng generalize traditional approach by including stochastic recovery, and by incorporating two economic states: solvency and insolvency.
- Analysis based on two continuous time stochastic processes for firm's asset value
  - Regime switching model with continuous sample paths
  - Diffusion model with jumps

Regime Switching Model

- Let $(X_t)_{t \geq 0}$ be the firm's asset value process that follows a diffusion process given by

$$dX_t = X_t \mu_{\epsilon(t)} dt + X_t \sigma_{\epsilon(t)} dW_t$$

  where W is a standard one-dimensional Brownian motion, and $(\epsilon(t))_{t \geq 0}$ is finite-state continuous-time Markov chain, independent of W, taking values $0, 1, \ldots, S-1$ with a known generator $(q_{ij})_{S \times S}$.

- We can think of these states as the credit rating of the firm (e.g. Aaa, Aa, ...)

- The simplest case is when $S = 2$, where $\epsilon(t) = 1, 0$ corresponds to healthy and default, respectively.
- Default is the random time τ given by

$$\tau = \inf\{t > 0 : \epsilon(t) = 0\}.$$

- Consistent with traditional reduced form models, default time has an intensity which is given by $\lambda_t = q_{\epsilon(t)0}$.

The Jump Diffusion Model

- Let W and ϵ be defined as a regime switching model.
- Let T_n be the nth jump of ϵ and let ξ be the jump amplitude of the firm's asset value at state i.
- The firm's asset value process X is assumed to satisfy

$$X_t = X_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \prod_{0<s\leq t, \Delta\epsilon(s)\neq 0} \xi_{\epsilon(s)}$$

where $\Delta\epsilon(t) \equiv \epsilon(t) - \epsilon(t-)$.

- To force downward pressure in the firm's asset value while in default, they assume $P(\xi_0 \geq 1) = 0$.

Default, Insolvency and Information

- **Default** Default occurs when a firm misses or delays a promised payment on one of its financial liabilities. Default *does not* imply the firm is insolvent, nor does it mean that the firm's debt will not be paid.

- **Insolvency** In default, there are two possible states of the firm, solvency and insolvency. Consistent with the structural approach to credit risk, once in default, insolvency occurs when the firm's asset value falls to a certain prescribed level x, i.e.,

$$\tilde{\tau} = \inf\{t > 0 : X_t < x, \epsilon(t) = 0\}.$$

- **Information Structure** Insolvency studied under two different filtrations: complete information and incomplete information.
  - Complete information corresponds to the information held by management, called investor $\mathcal{A}$.
  - The filtration structure for investor $\mathcal{A}$ is given by

$$\mathcal{F}_t^{\mathcal{A}} = \sigma\{X_s, \epsilon(s), 0 \leq s \leq t\} \vee \mathcal{N}$$

    where $\mathcal{N}$ is the collection of all null sets.

- Partial information corresponds to the information set held by the market, called investor $\mathcal{B}$.
- Investor $\mathcal{B}$'s filtration $\mathcal{F}_t^{\mathcal{B}}$ is the filtration generated by

$$1_{\{\tau \le t\}}, \sum_{n=0}^{\infty} X_{t_k} 1_{\{t_k \le t\}} \text{ and } \sum_{n=0}^{\infty} X_{T_n} 1_{\{T_n \le t\}}$$

for the regime switching model, and the filtration generated by

$$1_{\{\tau \le t\}}, \sum_{n=0}^{\infty} X_{t_k} 1_{\{t_k \le t\}}, \sum_{n=0}^{\infty} \xi_{\epsilon(T_n)} 1_{\{T_n \le t\}} \text{ and } \sum_{n=0}^{\infty} X_{T_n} 1_{\{T_n \le t\}}$$

for the jump diffusion model.

Insolvency Time (Regime Switching)

- Suppose τ is an $\mathbb{R}_+$ valued random variable with $F(t)$ its right continuous distribution function, and $F(t-) = P(\tau < t)$

- For a given filtration $\mathbf{J} = (\mathcal{J}_t, t \geq 0)$ the $\mathbf{J}$-adapted nonnegative process $(\lambda_s, s \geq 0)$ is the $\mathbf{J}$-intensity of τ *if*

$$\left(1_{\{\tau \leq t\}} - \int_0^{\tau \wedge t} \lambda_s ds, t \geq 0\right)$$

  is a $\mathbf{J}$-martingale

- If $N_t = 1_{\{\tau \leq t\}}$, then the process

$$M_t \equiv N_t - \int_0^{\tau \wedge t} \frac{dF(s)}{1 - F(s-)}$$

  is an $\sigma(N_u, u \leq t)$-martingale

Insolvency Time (Regime Switching)

Theorem 1 Under the augmented natural filtration $(\mathcal{F}_t^{\mathcal{A}})_{t\geq 0}$ of (X, ϵ), $\tilde{\tau}$ has a totally inaccessible component $\tilde{\tau}_\Lambda$, whose intensity, denoted as $d_t^{R,\mathcal{A}}$, is given by,

$$d_t^{R,\mathcal{A}} = 1_{\{\tilde{\tau}>t,\epsilon(t)\neq 0\}} q_{\epsilon(t)0}\Big(1_{\{X_t<x\}} + \frac{1}{2}1_{\{X_t=x\}}\Big).$$

Investor $\mathcal{B}$

Theorem 2 The insolvency time for investor $\mathcal{B}$ is totally inaccessible under filtration $\mathcal{F}_t(\tau)$. Moreover, if $t \in [t_k, t_{k+1})$, then when $\tilde{\tau} > t$, the insolvency intensity $d_t^{R,\mathcal{B}}$ of $\tilde{\tau}$ is

$$d_t^{R,\mathcal{B}} = \begin{cases} -\frac{\psi_t(\theta_0, t - t_k \vee T_n, \frac{1}{\sigma_0} \log \frac{x}{X_{t_k \vee T_n}})}{\psi(\theta_0, t - t_k \vee T_n, \frac{1}{\sigma_0} \log \frac{x}{X_{t_k \vee T_n}})}, \text{ if } \epsilon(t) = 0, \\ q_{\epsilon(t)0} \Phi\left(\frac{\frac{1}{\sigma_{\epsilon(t)}} \ln \frac{x}{X_{t_k \vee T_n}} - \theta_{\epsilon(t)} (t - T_n \vee t_k)}{\sqrt{t - T_n \vee t_k}} \right), \text{ if } \epsilon(t) \neq 0, \end{cases}$$

where $\theta_i = \frac{\mu_i}{\sigma_i} - \frac{\sigma_i}{2}$, Φ is the distribution function of standard normal random variable and

$$\psi(\theta, t, y) = P(\inf_{0 \leq s \leq t} W_s + \theta s > y).$$

Other Partial Information Models

- Duffie and Lando - Obscure the asset value process by adding independent noise. The observable is given by the discrete time process

$$Z_t = X_t + Y_t$$

where Y_t is the added noise process observed at times t_i for $i = 1 \dots \infty$

  - Market sees Z but not X.
  - Filtration of observable events: $(\mathcal{H}_t)_{t \geq 0} \subset \mathbf{F}$ where $\mathcal{H}_t = \sigma(Z_{t_i} : 0 \leq t_i \leq t)$

Çetin,Jarrow, Protter, and Yilidirim

- Alternative approach: Instead of adding noise, they take the market's information to be a strict subfiltration of the firm manager's information.
- Similar to structural model, they redefine the asset value to be the firms cash flow, now using the barrier $D = 0$.
- Modeler observes whether cash flows are positive or negative (i.e. the information set the market observes is $\mathcal{G}_t = \sigma\{sign(X_s) : s \leq t\}$)

- If $g(t) \equiv \sup\{s \leq t : X_s = 0\}$, then "potential default" begins at

  $$\tau_\alpha \equiv \inf\{t > 0 : t - g(t) \geq \frac{\alpha^2}{2} \text{ where } X_s < 0 \text{ for } s \in (g(t-), t)\}$$

- Default occurs at

  $$\tau_d = \inf\{t > 0 : X_t = 2X_{\tau_\alpha}, X_s < 0 \text{ for } s \in (\tau_\alpha, t)\}$$

- In this model, the default time is totally inaccessible, and the point process has an intensity, yielding an intensity based hazard rate model.

The Recovery Rate Process and Risky Debt Pricing

Motivation and Assumptions

- Default triggers the recovery rate process

- Introduction of insolvency time leads to probability of insolvency given default

- There exists a martingale measure P (not unique).
- Investor **A** has complete information, while investor **B** has only partial or delayed information.

The Traditional Approach prices risky debt prior to (or at) default

- Zero Coupon bond paying \$1 at time T if there is no default and \$$\tilde{\text{R}}$ if there is default before time T

$$V_C^i(t,T) = e^{-\int_t^T r(s)ds} E[\tilde{R}1_{\{\tau_o \leq T\}} + 1_{\{\tau_o > T\}} | F_t^i]$$

$$= e^{-\int_t^T r(s)ds}[1 - (1 - \tilde{R})P(\tau_o \leq T | F_t^i)] \text{ for } t \leq \tau_o$$

with $i \in \{\mathbf{A,B}\}$

Note: The difference in prices between investor **A** and **B** is quantified by the difference between the conditional probability of default before time T for investors **A** and **B**

Proposition 1 *Let* $D^{\mathbf{A}}(t,T)$ *and* $D^{\mathbf{B}}(t,T)$ *denote the conditional probability of default before time* T *under the natural filtration* $(F_t^{\mathbf{A}})_{t\geq 0}$ *and the delayed filtration* $(F_t^{\mathbf{B}})_{t\geq 0}$*, respectively. Suppose at time* t*, on the event* $\{\tau_o > t\}$, $\sigma(X_s) \subset F_t^{\mathbf{B}} \subset F_s^{\mathbf{A}}$ *where* $s < t$ *and* $(X_t)_{t\geq 0}$ *is the underlying Markov process, then*

$$D^{\mathbf{B}}(t,T)1_{\{\tau_o>t\}} = \frac{D^{\mathbf{A}}(s,T)}{D^{\mathbf{A}}(s,t)}1_{\{\tau_o>t\}}$$

The Recovery Rate Process

- the firm pays \$1 at time T if there is (and has been) no default,
- once default occurs ($\tau < T$), the firm either becomes insolvent, i.e., $\tau \leq \tilde{\tau} \leq T$; or stays solvent until T, i.e., $\tilde{\tau} > T$,
- if default occurs and the firm becomes insolvent before the debt's maturity T, then the bond pays a *realized* recovery rate of \$R, and
- if default occurs and the firm remains solvent up to the debt's maturity T, then the bond pays a fractional recovery rate of \$K where $R \leq K \leq 1$.

Proposition 2 *The time t value of the firm's zero-coupon bond to investor* **A** *or* **B** *is*

$$V^i(t,T) = e^{-\int_t^T r(s)ds} E_t^i[1 \cdot 1_{\{\tau > T\}} + K \cdot 1_{\{\tilde{\tau} > T, \tau \leq T\}} + R \cdot 1_{\{\tilde{\tau} \leq T, \tau \leq T\}}]$$

where $E_t^i = E[\cdot|F_t^i]$ is under a martingale measure with $i \in \{\mathbf{A}, \mathbf{B}\}$

Application of Proposition 2

- If the firm **has not** defaulted at by $t < \tau$, then

  $$V^i(t,T) = e^{-\int_t^T r(s)ds}[E_t^i[1_{\{\tau > T\}}] + (K-R)E_t^i[1_{\{\tilde{\tau} > T, \tau \leq T\}}] + RE_t^i[1_{\{\tau \leq T\}}]]$$

- If the firm **has defaulted** by time t, but the firm is still **solvent**, then

$$V^i(t,T) = e^{-\int_t^T r(s)ds}[(K-R)P(inf_{t \leq v \leq T} X_v > x|F_t^i) + R]$$

Key quantities to evaluate: the distribution of the default/insolvency times.

Calibration and Comparison

At the time of default, the two model prices are

$$V_C^i(\tau, T) = \tilde{R} e^{-\int_\tau^T r(s)ds}$$

and

$$V^{\mathbf{A}}(\tau, T) = V^{\mathbf{B}}(\tau, T) = \begin{array}{ll} R e^{-\int_\tau^T r(s)ds} & \text{if inso} \\ e^{-\int_t^T r(s)ds}[(K - R)P(inf_{t \le v \le T} X_v > x | F_t^i) + R] & \text{if sol} \end{array}$$

- $\tilde{R}$ Observe the (average) market prices for defaulted debt at time τ, denoted M_τ. Solve for $\tilde{R}$ by setting at time τ

$$M_\tau = \tilde{R} e^{-\int_\tau^T r(s)ds}$$

- **R** Observe the (average) market prices for defaulted debt at the time of emergence from financial distress, denoted M_∞. Set

$$R = M_\infty$$

- **K** Direct estimates of K are not available. However, we can implicitly estimate K, once we have M_τ and M_∞ by solving the following equation if the firm is still solvent.

$$M_\tau = e^{-\int_\tau^T r(s)ds}[(K - M_\infty)P(inf_{\tau \le v \le T} X_v > x | F_\tau^i) + M_\infty]$$

i.e.

$$\begin{aligned} K &= M_\infty + \frac{M_\tau e^{-\int_\tau^T r(s)ds} - M_\infty}{P(inf_{\tau \le v \le T} X_v > x | F_\tau^i)} \\ &= R + \frac{\tilde{R} - R}{P(inf_{\tau \le v \le T} X_v > x | F_\tau^i)} \end{aligned}$$

Then we have, prior to default ($\tau > T$)

$$V^i(t,T) - V_C^i = e^{-\int_t^T r(s)ds}(R - \tilde{R})P(\tau \leq T|F_t^i)$$
$$+e^{-\int_t^T r(s)ds}(K-R)P(\tilde{\tau} > T, \tau \leq T|F_t^i)$$

Applications to Correlated Defaults: A Numerical Interlude
taken from Jarrow, Gou, Zeng (March 9, 2005)

Problem How does investor **B**'s delayed information affect the estimate of two firms default correlation compared to investor **A**

Model Two firms who asset values follow the regime switching model.

$$dX_t^j = X_t^j \mu(\epsilon(t))dt + \sigma(\epsilon(t))X_t^j dW_j(t) \text{ j=1,2}$$

$\mu_i = -1, 2$, and $\sigma = 2, 4$ The underlying Brownian motions can be negatively, positively or have zero correlation.

W take a time interval of [0,1] with observation points at $0, \Delta, 2\Delta, \ldots, N\Delta = 1$ As $\Delta \to 0$ we get progressively more information, moving from case **B** to **A**

Numerical Interlude con't

Δt	negatively correlated B.M.	positively correlated B.M.	Independ
0.2000	-0.3455	0.5556	0.0136
0.1000	-0.4001	0.5740	-0.025
0.05	-0.4548	0.5700	0.0051
0.02	-0.4668	0.5676	0.0199
0.005	-0.4427	0.5825	0.0091
0.002	-0.4666	0.5789	0.0215
0.001	-0.4825	0.5856	-0.006

Table 1: Default correlation with respect to differently correlated Brownian motions