Consider a basket option of European type on N assets with payoff

$$V_T = \left[\sum_{n=1}^{N} a_n S_{n,T} - K \right]^+$$

Here a_n are the "weights" of the assets, $S_{n,T}$ is the price of the nth asset at the expiry T, and K is the strike price for the basket. The assets have a specified correlation ρ_{ij}. One approach to solve this pricing problem approximately is by replacing the arithmetic average of stocks by a geometric average and then by using the lognormality of the geometric average. This approach was introduced by [1], which is apparently based on ideas drawn from [2].

The quasi-Monte Carlo (QMC) method was used in [3] to estimate the price of this option. The parameters were $N = 2, \rho = 0.2, \sigma_1 = 0.2, \sigma_2 = 0.3$ (the last two are the volatilities), $a_1 = 0.6, a_2 = 0.4$. The QMC estimates were compared by the MC estimates. The presence of correlation does not add a significant complexity in simulation. The main problem is the generation of i.i.d normal random variables for assets in the basket. The required correlation between assets is simply obtained by evaluating an expression of the form $z_1\rho + z_2\sqrt{1-\rho^2}$. There is also a paper [8] that uses a variance reduction technique via MC to price basket options.

Basket options of Asian type were considered in [4]. The paper uses QMC methods with singular value decompositions and brownian bridge method. Numerical comparisons are given to compare a variety of simulation techniques. The parameter N takes the values $\{2, 4, 8, 16, 32, 64\}$.

There are several analytical methods based on estimating the untractable density function for the average of lognormal variables by a known density function. Three such methods are described and compared in [5]. The methods use the lognormal distribution and Edgeworth expansions, the inverse gamma distribution, and the Johnson distribution. The authors make the following statement in their abstract which I found interesting: "Although the price of such contracts can be obtained very accurately using Monte Carlo simulation, market participants prefer faster but less accurate approximations." The analytical approximations are compared with a 1 million sample based MC estimate, and based on this comparison the Johnson distribution is reported to win the race.

A different analytical approach introduced in [6] gives lower and upper bounds for the price of the basket option. These values are compared with MC and the geometric average approximation described in [1]. The paper also considers basket options with barrier.

The only paper [7] on American type basket options I could find (there is at least one paper on high-dim American options mentioned in the references that I have not looked at yet) uses "optimal quantization" to compute all of the conditional expectations. I am currently trying to understand what this method is about! In the numerical examples they use dimensions from 2 to 10. Interestingly assets are assumed to be independent: it turns out that this is the worst setting for this method, and the authors check the effectiveness of the method in this worst case scenario.

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