$n = 2 \qquad \Delta = \{\{1\},\{2\}\}$

2	7	3	12
12	9	1	22
4	8	0	12
18	24	4	

X_{00}	X_{01}	X_{02}	12
X_{10}	X_{11}	X_{12}	22
X_{20}	X_{21}	X_{22}	12
18	24	4	

TABLE ENTRY DATA SEC. PROBLEM

"Can X_{00} BE REAS. APPROX. FROM

min/max $\Big\{ X_{00} : X_{00} + X_{01} + X_{02} = 12$

$\vdots \qquad\qquad \vdots$

$X_{02} + X_{12} + X_{22} = 4 \qquad X_{ij} \geq 0,\ \text{integral} \Big\}$"

$$LP^-_0(b) = \min\{\ \text{"}\quad \text{"}\quad \text{"}\quad \text{"}\quad \underline{x} \in \mathbb{R}_{\geq 0}\}$$

$$\leq IP^-_0(b) = \min\{x_0 : A_\Delta \underline{x} = \underline{b},\ \underline{x} \in \mathbb{N}\}$$

$$\leq \text{true value } x_0$$

$$\leq \max\{x_0 : \text{"} \quad \text{"}\} = IP^+_0(b)$$

$$\leq \max\{\ \text{"} \quad \text{"}\ \underline{x} \in \mathbb{R}_{\geq 0}\} = LP^+_0(\underline{b})$$

x_0 is secure if disc. between IP^- & IP^+ large.

<u>Idea</u> Use LP^- instead of IP^-

<u>note</u> 1) A_Δ is $\{0,1\}$ matrix

2) <u>LOTS</u> of problems from comb. opt. when $IP^- = LP^-$

- these matrices are $\{0,1\}$ w/ $\{0,1\}$ cost vectors.

A measure of discrep. b/ween LP^- & IP^-

$$\text{gap}^-_\Phi(\underline{b}) = \text{opt value of } IP^-(\underline{b}) - \text{opt value of } LP^-(\underline{b})$$

$$\text{gap}^-_\Phi := \max\{\text{gap}(\underline{b}) : \underline{b} \in \mathbb{N}A_\Delta\}$$

Hochster: compute gap_Δ^- algeb.

Sull: instances where Δ is on n vertices

& $\mathrm{gap}_\Delta^- \geq 2^{n-3} - 1$.

$M^- = \langle \underline{x}^{\underline{u}} : \underline{u}$ non-opt sol^n to $IP^-(A_u)\rangle$

= irred. decomp. of M^-

$= \bigcap_{\text{finite}} \left| \langle x_i^{\gamma_i} : i \in \sigma \subseteq [\cancel{n}\, 2^n] \rangle \right|$

$$g_y(I_{A_\Delta}) = \{ x^{\underline{u}} - x^{\underline{v}} \}$$

with $u_0 \geq V_0$

if $u_0 > V_0 = 0$

$\quad \| $

$\quad \alpha$

$$gap^- \geq \alpha - 1 \geq 1$$

Q

~~Problem~~ perhaps this happens for very few $\underline{b}$'s

i.e.

$$\frac{\#\{\underline{b}\text{'s} : gap^-(b) \geq 1\}}{\text{all margins } \underline{b}} = 0$$

Problem

Construct Δ w/ a Markov element

$$x_a^{\alpha}\, \underline{x}^{\underline{u}'} - \underline{x}^{\underline{v}'} \quad \text{s.th.}$$

1) $\alpha \geq 2$ 2) $|\mathrm{supp}(u') \cup \mathrm{supp}(v')| \geq \dim(\Delta)$